LQ-problem

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Publication date:
1988

Citation for published version (APA):
Engwerda, J. C. (1988). *LQ-problem: The discrete-time time-varying case*. (Research memorandum / Tilburg University, Department of Economics; Vol. FEW 359). Unknown Publisher.

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RESEARCH MEMORANDUM

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LQ-PROBLEM: THE DISCRETE-TIME
TIME-VARYING CASE

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FEW 359

LQ-PROBLEM: THE DISCRETE-TIME TIME-VARYING CASE

Abstract

In this paper we solve the Linear Quadratic (LQ) regulator problem for discrete-time time-varying systems. By making an appropriate state-space decomposition of the system, sufficient conditions are derived under which this LQ-problem is solvable and, moreover, the closed-loop system becomes exponentially stable.

These conditions are extensions of the time-invariant notions of stabilizability and detectability. Unfortunately, in general these conditions are not necessary. The approach we take provides, however, also a good insight into the difficulties that occur if one looks for both necessary and sufficient conditions solving the problem.

Keywords

Linear discrete-time time-varying systems, stabilizability, detectability, state-space decomposition.

I. INTRODUCTION

In the past much research has been done on the subject under which conditions the linear quadratic regulator problem has a solution if the considered system is time-varying, see e.g. Kwakernaak et al (1971), Hager et al (1976) and Anderson et al (1980, 1981).

In Anderson et al (1981) it was claimed that under a uniform stabilizability and uniform detectability condition the Kalman filter, the dual of the LQ problem, is exponentially stable (under the usual system noise assumptions). Engwerda showed by means of a counterexample in (1988-a), however, that this claim is not correct. In fact he showed that the definitions given by Anderson et al of uniform stabilizability and uniform detectability do not imply that the system is stabilizable and detectable, respectively.

For that reason Engwerda formulated in the same paper new conditions which imply (exponential) stabilizability and detectability of the system. These new conditions are formulated in terms of a transformed system that is obtained by applying an appropriate state-space decomposition. In this paper we use and extend this analysis in order to obtain sufficient conditions under which the LQ-problem has a solution with the property that it makes the closed-loop system exponentially stable. The paper is organized as follows.

First, we introduce in section 2 the notions of uniform periodic smooth exponential stabilizability and detectability respectively. Then, we show in section 3 that under these conditions the LQ problem has a solution. Consecutively, we show that when the resulting optimal state-feedback control is applied the system becomes exponentially stable.

The paper ends with some concluding remarks.

II. PRELIMINARIES

As announced in the previous section we introduce here the notions of uniform periodic smooth exponential stabilizability and detectability. To that extent we first state the considered system and some definitions. We will be dealing with the following linear time-varying discrete-time system:

$$\Sigma_y: \begin{array}{lll} x(k+1) & = & A(k)x(k)+B(k)u(k);\ x(k_0)=\bar{x} \\ y(k) & = & C(k)x(k), \end{array}$$

where $x(k)\in\mathbb{R}^n$ is the state of the system, $u(k)\in\mathbb{R}^m$ the applied control and $y(k)\in\mathbb{R}^r$ the output at time k. Moreover, we assume that all matrices A(.), B(.) and C(.) are bounded.

Since the system is time-varying it is convenient to have the notation:

Let N be any positive number, then

$$\begin{array}{lll} A(k+N,k) & := A(k+N-1)\ .....\ A(k) & \text{if } N\geq 1 \\ & := I & \text{if } N=1 \\ S[k,k-N] & := [B(k)|A(k+1,k)B(k-1)|\dots|A(k+1,k-N+1)B(k-N)] & \\ W[k,k-N] & := [C(k)^T|\dots\dots|\{C(k+N)A(k+N,k)\}^T]^T & \\ v[k,\ell] & := (v^T(k),\dots\dots,v^T(\ell))^T & \\ v[k,.] & := (v^T(k),v^T(k+1,\dots\dots)^T & \end{array}$$

$x(k,k_0,\bar{x},u)$ is the state of the system at time k resulting from the initial state $\bar{x}$ at time k_0 if the input $u[k_0,k-1]$ is applied

$y(k,k_0,\bar{x},u) := C(k)x(k,k_0,\bar{x},u)$. □

Using this notation we can give now easily formal definitions of several notions that are used later on in this paper.

Definition 1

The initial state $\bar{x}$ of the system Σ_y is said to be

* exponentially stable at k_0 if there exist positive constants α and M such that $\|x(k,k_0,\bar{x},0)\|\leq Me^{-\alpha(k-k_0)}\|\bar{x}\|$ for any $k>k_0$
* exponentially stabilizable at k_0 if there exists a control sequence $u[k_0,.]$, with the property that u(.) converges exponentially fast to

zero, and positive constants α and M such that
$\|x(k,k_0,\bar{x},u)\| \leq Me^{-\alpha(k-k_0)}\|\bar{x}\|$ for any $k>k_0$.

* unobservable at k_0 if $y(k,k_0,\bar{x},0)=0$ for any $k \geq k_0$.
* exponentially detectable at k_0 if there exists a finite integer $N>0$ such that $\bar{x}$ modulo $X_e^-(A(.,k_0))$ is determined from any $y[k_0,k_0+N-1]$ and $u[k_0,k_0+N-2]$. Here $X_e^-(A(.,k_0))$ is the linear subspace consisting of all exponentially stable states at time k_0.

Like all exponentially stable states, the set of all unobservable states at k_0 constitute a linear subspace. We denote it by U_{k_0}. Now, Σ_y is called observable at k_0 if x=0 is the only unobservable state at k_0. Moreover, we say that Σ_y is exponentially stable (respectively stabilizable, exponentially detectable) at k_0 if any initial state of Σ_y possesses the corresponding property at k_0.
Using these concepts the notion of uniform exponential stabilizability and detectability are defined as follows.

<u>Definition 1 (continued):</u>
Σ_y is called uniformly exponentially stabilizable (respectively detectable) if Σ_y has the corresponding property at any time $k \geq k_0$ and, moreover, the constants $\alpha(k)$ and $M(k)$ appearing in the definitions satisfy the inequalities $\alpha(k) \geq \bar{\alpha} > 0$ and $M(k) \leq \bar{M} < \infty$ for some $\bar{\alpha}$ and $\bar{M}$. □

Another notion that plays an important role in our analysis is the concept of reachability. Formally, we call a state $\bar{x}$ reachable (from zero) if there exists a control sequence $u[N,k-1]$ with $-\infty<N<k$ such that $x(k,N,0,u)=\bar{x}$. The linear subspace of all reachable states at time k is denoted by R_k.
Now, Engwerda showed in (1987) that R_k is A(k)-invariant, that is, $A(k)R_k \subset R_{k+1}$. Moreover, he showed in (1988-b) that the unobservability subspace has this property too. These properties are used in lemma 2, where we give an equivalent system representation of Σ_y. To that extent we first introduce the state-space decomposition

$$X_1(k) \qquad = R_k \cap U_k;$$

$$X_1(k) \oplus X_2(k) = R_k;$$
$$X_1(k) \oplus X_2(k) \oplus X_3(k) = \mathbb{R}^n,$$

where X_1, X_2 and X_3 are chosen orthogonal. According to Engwerda (1988-a) the next result holds

Lemma 2

There exists an orthogonal state-space transformation $x(.)=T'(.)x'(.)$, which does not effect the boundedness property of the system parameters, such that Σ_y is described by the recurrence equation.

$$\Sigma_y': \begin{pmatrix} x_1'(k+1) \\ x_2'(k+1) \\ x_3'(k+1) \end{pmatrix} = \begin{pmatrix} A_{11}'(k) & A_{12}'(k) & A_{13}'(k) \\ 0 & A_{22}(k) & A_{23}'(k) \\ 0 & 0 & A_{33}'(k) \end{pmatrix} \begin{pmatrix} x_1'(k) \\ x_2'(k) \\ x_3'(k) \end{pmatrix} + \begin{pmatrix} B_1'(k) \\ B_2'(k) \\ B_3'(k) \end{pmatrix} u'(k)$$

$$y(k) = (0 \quad C_1'(k) \quad C_2'(k))x'(k) ,$$

where

Σ_1': $x_1'(k+1) = A_{11}'(k)x_1'(k) + B_1'(k)u'(k)$ is reachable at any time $k \geq k_0$.

Σ_2': $x_2'(k+1) = A_{22}'(k)x_2'(k) + B_2'(k)u'(k) + A_{23}'(k)A_{33}'(k+1,k_0)x_3'(k_0)$
$y(k) = C_2'(k)x_2'(k)$ is both reachable and observable at any time $k \geq k_0$;

Σ_3': $x_3'(k+1) = A_{33}'(k)x_3'(k)$. □

In order to obtain sufficient conditions for exponential stabilizability and detectability of Σ_y at k_0 we introduce the notions of periodic smooth controllability and observability. Roughly spoken, we say that a system is periodically smoothly controllable if there exists a finite time period such that whenever such a time period has passed, the system has been at least once controllable during that period.

Definition 3.

Σ_y is called periodically smoothly controllable at k_0 if there exist positive constants ε and k_1 such that for all $k>0$ there exists an integer $k_2(k)$ in the interval $[k_0+(k-1)\ k_1,\ k_0+k\ k_1]$ for which
$S[k_2-2\ k_1,k_2]S^T[k_2-2\ k_1,k_2] \geq \varepsilon\ I$.

Similarly we say that Σ_y is periodically smoothly observable at k_0 if there exist positive constants b and k_1 such that for all $k>0$ there exists an integer $k_2(k)$ in the interval $[k_0+(k-1)\ k_0,\ k_0+k\ k_0]$ for which $W[k_2,k_2+2\ k_1]\ W^T[k_2,k_2+2\ k_1] \geq b\ I$.
Instead of periodic smooth controllability (respectively observability) of Σ_y we often use the phraseology periodic smooth controllability of the pair (A(.), B(.)) and observability of the pair (C(.), A(.)), respectively. □

With the notation of lemma 2, we then have as a special case from theorem 20 of Engwerda (1988-a):

Theorem 4:
Σ_y is both exponentially stabilizable and exponentially detectable at k_0 if the following three conditions are satisfied:

i) Σ_1' is uniformly exponentially stable;

ii) Σ_2' is both periodically smoothly controllable and observable at k_0;

iii) Σ_3' is exponentially stable at k_0.

In the next section we will need that Σ_y is both uniformly exponentially stabilizable and uniformly exponentially detectable. From theorem 4 it is clear that this property holds if additional to the conditions i) and ii), Σ_3' is uniformly exponentially stable. We state this result in a corollary.

Corollary 5:
Σ_y is both uniformly exponentially stabilizable and uniformly exponentially detectable if

i) Σ_1' is uniformly exponentially stable

ii) Σ_2' is both periodically smoothly controllable and observable at k_0

iii) Σ_3' is uniformly exponentially stable.

In the sequel we call conditions i) upto iii) in corollary 5 the exponential stabilizability and detectability (E.S.D.) conditions. Note that for time-invariant systems these three conditions are necessary too.

III <u>The solution of the LQ control problem.</u>

In this section we consider the LQ optimal control problem:

(1) $\min\limits_{u[k_0,.]} \lim\limits_{N\to\infty} J_N$, subject to Σ_y

where

$$J_N = \sum_{k=k_0}^{k_0+N-1} \{\|y(k)\|^2 + \|u(k)\|^2_{R(k)}\} + \|y(k_0+N)\|^2.$$

In the sequel we take without loss of generality $k_0 = 0$. Moreover shall C^TC be denoted by Q.

Furthermore, we assume that the following, the so-called Sufficient Control Existence (S.C.E.), conditions are satisfied.

(S.C.E.)

i) The E.S.D. conditions of corollary 5

ii) a) $R(k) \geq \beta I$ for some $\beta > 0$, for all $k \geq 0$

or b) $B^T(k)Q(k+1)B(k) \geq \beta_1 I$, $Q(k) \geq \beta_2 I$ and $R(k) \geq 0$ for some $\beta_i > 0$, $i = 1,2$ for all k.

We will show that under these conditions an optimal control for the LQ problem exists and is given by:

(2) $u(k) = - F(k)\, x(k)$

where $F(k) = (R(k) + B^T(k)K(k+1)B(k))^{-1} B^T(k)K(k+1)A(k)$,

and K(k) is given by

$K(k) = \lim\limits_{N\to\infty} K_N(k)$, where $K_N(k)$ is obtained from the

recursive equation:

(RRE): $K_N(k) = A^T(k)\{K_N(k+1) - K_N(k+1)B(k)(R(k) + B^T(k)K_N(k+1)B(k))^{-1}.$

$$. B^T(k)K_N(k+1)\}A(k) + Q(k),\ K_N(N) = Q(N).$$

Moreover, we show that if this optimal feedback controller (2) is used to regulate the system, the closed-loop system becomes exponentially stable.

Theorem 6:

Let Σ_y satisfy the S.C.E. conditions.
Then, controller (2) minimizes $\lim_{N \to \infty} J_N$.

Proof:

First, consider the optimal control problem $\min_{u[0,N-1]} J_N$ subject to Σ_y.
The optimal control for this problem is:

(3) $\quad u_N(k) = - F_N(k)\, x(k)$

where $F_N(k) = (R(k) + B^T(k)K_N(k+1)B(k))^{-1}\, B^T(k)K_N(k+1)A(k)$, and $K_N(k)$ is given by the resursive equation (RRE).
Moreover, we have that the corresponding minimal control cost equals $\bar{J}_N := x^T(0)K_N(0)x(0)$ (see e.g. Bertsekas (1976)).
Since, due to our assumptions, Σ_y is exponentially stabilizable we have that there exists a control sequence such that $\lim_{N\to\infty} J_N$ remains finite.
Now, $\bar{J}_N$ is a monotonically increasing sequence.
Consequently, $\lim_{N\to\infty} K_N(0)$ exists. Moreover, since Σ_y is uniformly exponentially stabilizable, a similar reasoning shows that $\lim_{N\to\infty} K_N(k)$ exists for any k.
So, we have shown now that $\lim_{N\to\infty} u_N(k)$ exists. Denote this limit by $\bar{u}(k)$.
Due to the monotonicity property of $\bar{J}_N$ we can apply Bellman's principle to conclude that

$$\lim_{N\to\infty} \bar{J}_N \leq \min_{u[0,.]} \lim_{N\to\infty} J_N.$$

So, the only thing left to be proved is that

$$\lim_{N\to\infty} \bar{J}_N \geq \min_{u[0,.]} \lim_{N\to\infty} J_N.$$

This can be done by using some elementary analysis. Since J_N consists of the sum of positive functions, Fatou's lemma (see Rudin (1976), Theorem 11.31) can namely be applied to conclude that the order of taking limits and summations can be interchanged (for a more detailed proof see Engwerda (1988-b)). This completes the proof. ▯

To prove exponential stability of $x(k+1) = (A-BF)(k)x(k)$ we need an extended result of Lyapunov's lemma. This result can be proved along the lines the proof of the corresponding property for uniformly stabilizable and detectable systems in Anderson et al (1981) (see Engwerda (1988-b) for a detailed proof).

Lemma 7: (Extended lemma of Lyapunov).
Let $A(.)$ and $H(.)$ be bounded.
Suppose that $(A(.), H(.))$ is periodically smoothly observable and that there is a bounded positive semi-definite symmetric matrix sequence $P(k)$ satisfying $A^T(k)P(k+1)A(k) - P(k) = -H^T(k)H(k)$ on $[0,\infty)$.
Then $x(k+1) = A(k)\, x(k)$ is exponentially stable. ▯

We are now able to prove the main result of this paper.

Theorem 8:
Let the S.C.E. conditions be satisfied.
Then there exist constants $M<\infty$ and $\alpha > 0$ such that $\|(A-BF)(k,0)\| \leq Me^{-\alpha k}$.

Proof:
We know from theorem 6 that we can associate the following control problem with $(A-BF_N)(.)$:

$$\min_{u[0,\, N-1]} J_N, \text{ subject to } \Sigma_y.$$

We reconsider this minimization problem.
From lemma 2 we have that this problem can be rewritten as:

$$\min_{u'[0,N-1]} \sum_{k=0}^{N-1} \{\| x_2'(k) \|^2_{C_2'^T(k)\, C_2'(k)} + \| x_3'(k) \|^2_{C_3'^T(k) C_3'(k)} +$$

$$\| u'(k) \|^2_{R(k)}\} + \|x_2'(N)\|_{C_2'^T(N)C_2'(N)} + \|x_3'(N)\|^2_{C_3'^T(N)C_3'(N)},$$

subject to Σ_y.

According to the proof of theorem 6 the optimal control is given by (3). Substitution of the system parameters yields (by induction on k) that $K_N'(k)$ has the following structure:

$$K_N'(k) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_{22} & K_{23} \\ 0 & K_{23} & K_{33} \end{bmatrix}_N' (k) \qquad \text{(i)}$$

and consequently

$$F_N'(k) = (0 \mid F_2 \mid F_3)_N'(k) \qquad \text{(ii)}$$

Since $K_N'(.)$ converges to $K'(.)$ and $F_N'(.)$ to $F'(.)$ it is clear that $K'(.)$ and $F'(.)$ have the structure of (i) and (ii), respectively. Since $K'(.)$ converges for any k we have from (RRE), moreover, that $K'(.)$ satisfies the recurrence equation

$$K'(k) = A'^T(k)\{ K'(k+1) - K'(k+1)B'(k)(R(k) + B'^T(k)K'(k+1)\, B'(k))^{-1} B'^T(k)K(k+1)\}A'(k) + Q'(k),$$

which can be rewritten as:

$$K'(k) = (A-BF)'^T(k)\; K'(k+1)\; (A-BF)'(k) + (Q+F^TRF)'(k).$$

In particular it follows now, by substitution of all the system parameters, that

$$K_{22}'(k) = (A_{22}-B_2F_2)'^T(k)\; K_{22}'(k+1)\; (A_{22}-B_2F_2)'(k) + (C_2^TC_2 + F_2^TRF_2)'(k)$$

with

$$F_2'(k) = (R(k) + B_2'^T(k)K_{22}'(k+1)B_2'(k))^{-1}B_2'^T(k)K_{22}'(k+1)A_{22}'(k).$$

From the S.C.E. conditions it follows that $K'_{22}(.)$ and $F'_2(.)$ are bounded. Now, let

$$D' := (C_2'^T \mid F_2'^T R^{\frac{1}{2}})^T$$

then,

$$A'_{22} - B'_2F'_2 = A'_{22} - [0 \mid B'_2] R^{-\frac{1}{2}} D' \text{ and } C_2'^T C'_2 + F_2'^T R F'_2 = D'^T D'.$$

Since the observability property of (A'_{22}, C'_2) implies that (A'_{22}, D') has the same property, it is easily shown that $(A'_{22}-B'_2F'_2, D')$ is periodically smoothly observable too (see e.g. Anderson (1981) or Engwerda (1988-b)). Application of lemma 7 yields now that $(A_{22}-B_2F_2)'(.)$ is exponentially stable.

Since the feedback gain F' does not influence the exponential stability of Σ'_1 and Σ'_3, we conclude that the overall CL-system, (A-BF)'(.), is exponentially stable.

Finally, we note that since the transformation matrix T'(k) is bounded, exponential convergence of x'(k) implies that the same property holds for the original state of the system x(k). Which completes the proof. $\square$

Concluding Remarks.

In this paper we solved the discrete-time time-varying LQ optimal control problem under some weak conditions on the system. These conditions were formulated in terms of a transformed system that was obtained by making use of several invariance properties of the system.
A major problem occurring was to find a suitable state-space representation. This, since the prerequisite that the convergence properties of the transformed and original system must coincide, reduces the class of admissible transformations.
Fortunately, we succeeded in finding such a transformation which, moreover, was useful when we had to prove that the closed-loop system is exponentially stable if the optimal LQ control is applied.
An interesting question which remains to be solved is whether the LQ-problem has also an exponentially stabilizing solution when the system is both uniformly exponentially stabilizable and detectable (in the sense defined here). A subsequent question, which immediately arises if the answer to the previous one is affirmative, is then to give both necessary and sufficient conditions (that can a priori be verified) that guarantee this uniform exponential stabilizability and detectability.
Last, but not least, we note that the obtained results can be used in a straightforward manner to solve e.g. the LQG-problem, the EQL-problem (see Engwerda (1988-b)) and the Kalman filter problem for discrete-time time-varying systems.

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